Conyers Sixth Form Transition Work

A Level Further Mathematics

Congratulations on your enrolment to Conyers Sixth Form; please find below, tasks that will aid your transition from GCSE to Level 3 study. Your subject teacher will check completion of this work in September.

1. Open the accompanying Year 12 Further Maths Transition Booklet and read through the example for each topic.

2. Complete the questions for the topics, ensuring to set out your work neatly. Do the work in sections and spread it out over a number of days.

3. If you need addition support, click the link on the first page and watch the videos on the appropriate topic.

Transition Work for Further Maths

Completing the Square

Example

Consider $x^2 - 2x$

Adding 1 gives $x^2 - 2x + 1$

Now $x^2 - 2x + 1 \equiv (x - 1)^2$ which is a perfect square.

Adding the number 1 was not a guess, it was found by using the fact that

$$x^2 + 2ax + \boxed{a^2} \equiv (x + a)^2$$

We see from this that the number to be added is always (half the coefficient of x)^2.

Hence $x^2 + 6x$ requires 3^2 to be added to make a perfect square,

i.e. $x^2 + 6x + 9 \equiv (x + 3)^2$

To complete the square when the coefficient of x^2 is not 1, we first take out the coefficient of x^2 as a factor,

e.g. $2x^2 + x \equiv 2(x^2 + \frac{1}{2}x)$

Now we add $(\frac{1}{2} \times \frac{1}{2})^2$ inside the bracket, giving

$$2(x^2 + \tfrac{1}{2}x + \tfrac{1}{16}) \equiv 2(x + \tfrac{1}{4})^2$$

Take extra care when the coefficient is negative

e.g. $-x^2 + 4x \equiv -(x^2 - 4x)$

Then $-(x^2 - 4x + 4) \equiv -(x - 2)^2$, therefore $-x^2 + 4x - 4 \equiv -(x - 2)^2$

(a) Write $x^2 - 6x + 1$ in the form $(x + a)^2 + b$ where a and b are integers.

……………………………………………………

(2)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = x^2 - 6x + 1$

……………………………

(1)

(a) Write $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$ where a, b, and c are integers.

……………………………………………………

(3)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 - 12x + 23$

……………………………

(1)

Simplifying Algebraic Fractions

Worked Example

Simplify $\frac{4x+6}{2x^2-7x-15}$

1. *Top* : $4x + 6 = 2(2x + 3)$

Use the fact that there is a factor of $(2x + 3)$ *to help factorise the bottom* :

Bottom : $2x^2 - 7x - 15 = (2x + 3)(x - 5)$

so $\frac{4x+6}{2x^2-7x-15} = \frac{2(2x+3)}{(2x+3)(x-5)}$

2. $= \frac{2}{(x-5)}$

Simplify $\frac{4(x + 5)}{x^2 + 2x - 15}$

Simplify fully $\frac{x^2 + 3x - 4}{2x^2 - 5x + 3}$

Adding/Subtracting Algebraic Fractions

Before fractions can be added or subtracted, they must be expressed with the same denominator, i.e. we have to find a common denominator. Then the numerator can be added or subtracted.

Example 1: Simplify $\frac{2}{p} + \frac{3}{q}$

Solution:

$$\frac{2}{p} + \frac{3}{q} = \frac{2q}{pq} + \frac{3p}{pq} = \frac{2q + 3p}{pq}$$

Example 2: Simplify $\frac{1}{x} + \frac{x}{x-1}$

Solution:

Find the lowest common denominator. $x(x-1)$

Write each fraction in terms of the lowest common denominator $\Rightarrow$ $\frac{1}{x} = \frac{x-1}{x(x-1)}$

$$\frac{x}{x-1} = \frac{x^2}{x(x-1)}$$

Simplify $\Rightarrow \frac{1}{x} + \frac{x}{x-1} = \frac{x-1}{x(x-1)} + \frac{x^2}{x(x-1)} = \frac{x^2+x-1}{x(x-1)}$

Multiplying and Dividing Algebraic fractions

Worked Example

Divide $\frac{x+3}{x-4}$ *by* $\frac{2x+6}{x^2-16}$, *giving your answer as a simplified fraction.*

First "flip'n'times" :

$$\frac{x+3}{x-4} \div \frac{2x+6}{x^2-16} = \frac{x+3}{x-4} \times \frac{x^2-16}{2x+6}$$

Now follow rules for multiplying :

1. $$\frac{x+3}{x-4} \times \frac{x^2-16}{2x+6} = \frac{x+3}{x-4} \times \frac{(x-4)(x+4)}{2(x+3)}$$

Cancel the $(x+3)$ *s and the* $(x-4)$ *s :*

2&3. $$= \frac{1}{1} \times \frac{1\times(x+4)}{2\times 1}$$

$$= \frac{x+4}{2}$$

4. *No need to factorise and cancel again as that was done in* 1.

$$\frac{j^2-2j+1}{j^2-1} \times \frac{j^2+4j+3}{j^2+2j-3}$$

$$\frac{s^2+4s-5}{s^2+s-2} \div \frac{s^2+10s+25}{s^2+8s+15}$$

Sine and Cosine Rule

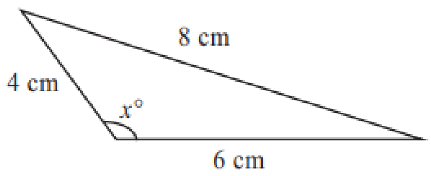


Diagram **NOT** accurately drawn

Calculate the value of x.
Give your answer correct to 1 decimal place.

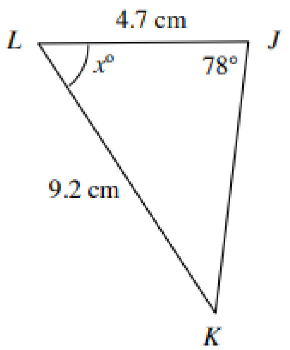


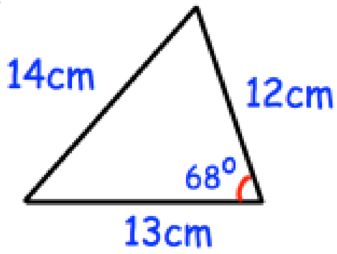
Diagram **NOT** accurately drawn

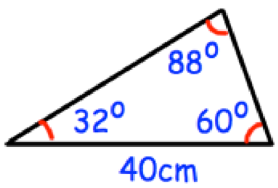
Calculate the value of x.
Give your answer correct to 1 decimal place.

A triangle has sides of length 4 cm, 6 cm and 8 cm.
Calculate the size of the largest angle in this triangle.

Give your answer correct to 1 decimal place.

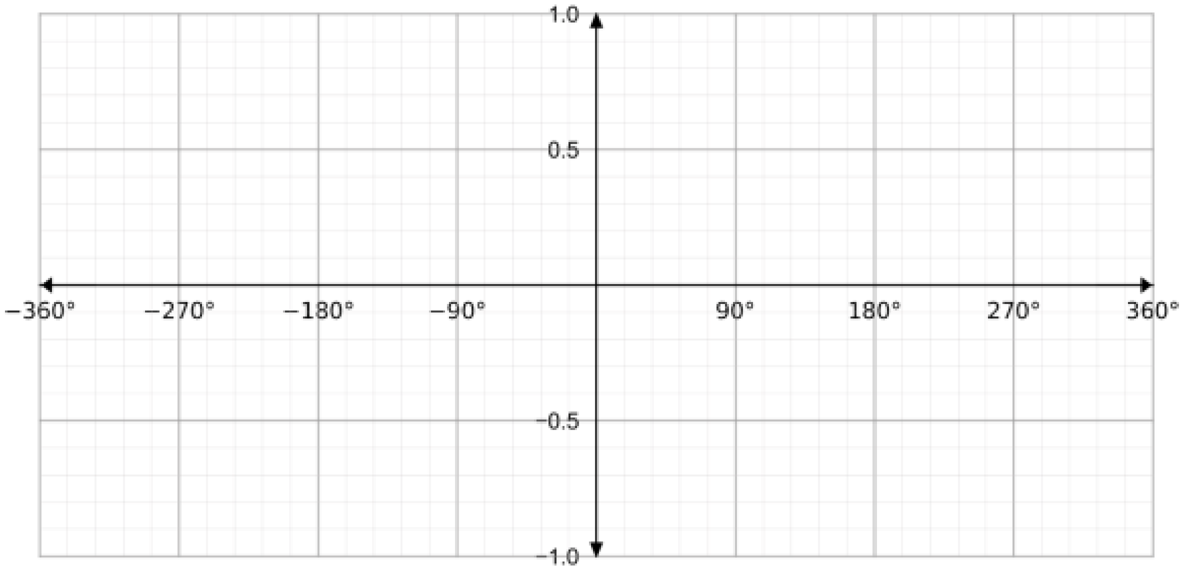
Area of a Triangle





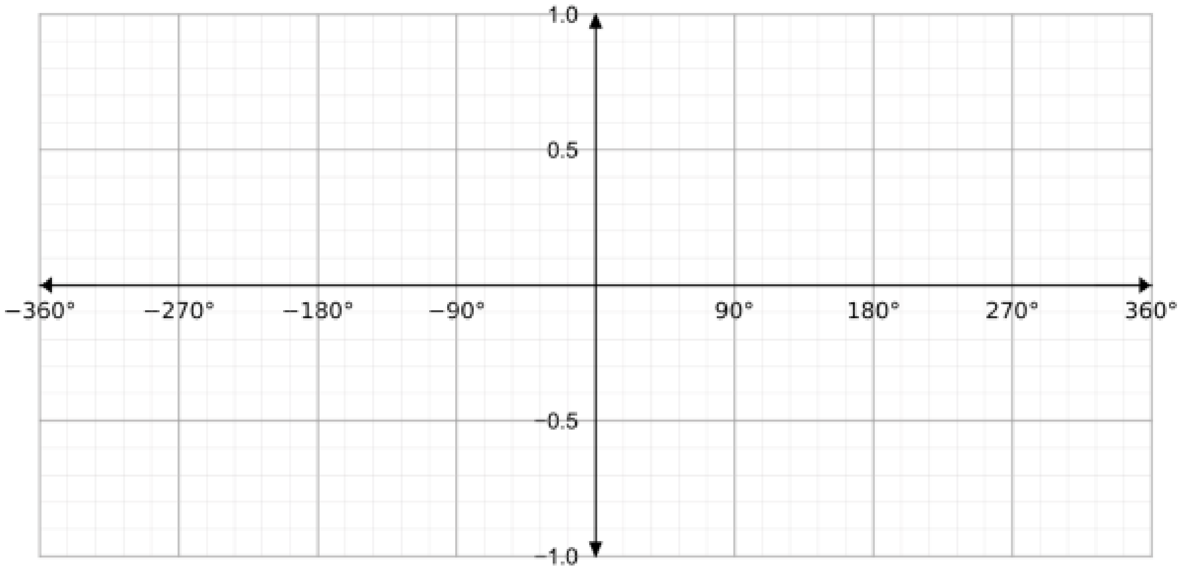
Sketch the graph of $y = \sin(x)$ on the axes below for the region $-360 \leq x \leq 360$. *(Level 5)*

[2 marks]

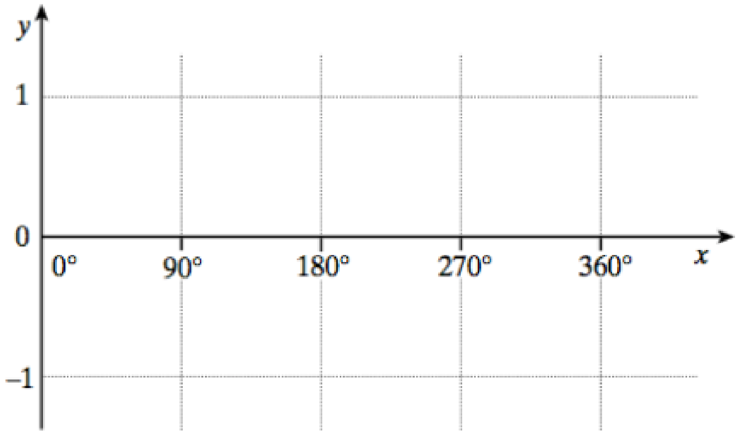


Sketch the graph of $y = \cos(x)$ on the axes below for the region $-360 \leq x \leq 360$. *(Level 5)*

[2 marks]



(a) Sketch the graph of $y = \cos x$ for $0 \leq x \leq 360$.



One solution of the equation $\cos x = 0.766$ is $x = 40°$

(b) Find the other solution of this equation for $0 \leq x \leq 360$.

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(2)

(c) Use your sketch to work out the value of cos 200°

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(1)

(a) Write $x^2 - 6x + 1$ in the form $(x + a)^2 + b$ where a and b are integers.

$(x - 3)^2 - 9 + 1$

$(x - 3)^2 - 8$

$(x - 3)^2 - 8$

(2)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = x^2 - 6x + 1$

$(3, -8)$

(1)

(a) Write $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$ where a, b, and c are integers.

$2(x^2 - 6x + 11.5)$

$2((x - 3)^2 - 9 + 11.5)$

$2((x - 3)^2 + 2.5)$

$2(x - 3)^2 + 5$

$2(x - 3)^2 + 5$

(3)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 - 12x + 23$

$(3, 5)$

(1)