Conyers Sixth Form Transition Work

A Level Mathematics

Congratulations on your enrolment to Conyers Sixth Form; please find below, tasks that will aid your transition from GCSE to Level 3 study. Your subject teacher will check completion of this work in September.

1. Open the accompanying Year 12 Maths Transition Booklet and read through the example for each topic.

2. Complete the questions for the topics, ensuring to set out your work neatly. Do the work in sections and spread it out over a number of days.

3. If you need addition support, click the link on the first page and watch the videos on the appropriate topic.

A Level Maths and Further Maths Summer Work

The starting point for A Level Maths assumes strong algebra skills from Higher tier GCSE Maths. This means you will need to keep practising these skills over the summer so that you don't forget them in September!

Over the summer holidays you will need to complete this booklet. To help you complete it, you should use your GCSE class books as well as visiting the website

https://www.cimt.org.uk/projects/mepres/step-up/index.htm which contains explanations and questions to try.

You can also use MathsWatch to help you. Centre ID : conyers
Username : conmaths
Password : square

or you could use the "Getting Ready for A Level Maths" videos from this website

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When you arrive back at Conyers in September, you must have the completed booklet with you. The booklet will only be considered completed if:

- Every question has been attempted.
- There is clear, well presented working for each question – **NOT just a list of answers!**

You will be tested on these topics in your first lesson.

Make sure you contact a member of staff **before the end of term** if you need any help with these topics

Just to get warmed up!

Example

Simplify $5x - 3(4 - x)$

$$5x - 3(4 - x) = 5x - 12 + 3x$$
$$= 8x - 12$$

NOTE: If there is no coefficient (number) in front of a variable it is understood to be 1.
Be careful to take note of the operator (plus or minus) in front of the term.

Simplify the following

1. $2x^2 - 4x + x^2$

2. $5a - 4(a + 3)$

3. $8pq - 9p^2 - 3pq$

4. $2(a^2 - b) - a(a + b)$

5. $3 - (x - 4)$

6. $5x - 2 - (x + 7)$

7. $(3y^2 + 4y - 2) - (7y^2 - 20y + 8)$

1 Simultaneous Equations

You need to be able to solve systems of two linear simultaneous equations, e.g.

$$2x + 5y \; = \; 23 \quad -3x + 2y \; = \; -6$$

You might be used to doing this by the **method of equal coefficients** (this means making the same amount of x or y in each equation, then adding or subtracting).

You need to be able to solve linear simultaneous equations by the **method of substitution**, e.g.

<u>Example</u>

Make x the subject in the first equation:

$$2x + 5y = 23$$

$$2x = 23 - 5y$$

$$x = \frac{23-5y}{2}$$

Substitute into the second equation:

$$-3x + 2y = -6$$

$$-3\left(\frac{23-5y}{2}\right) + 2y = -6$$

$$-3(23 - 5y) + 4y = -12$$

$$-69 + 15y + 4y = -12$$

$$-69 + 19y = -12$$

$$19y = 57$$

$$y = 3$$

Now work out what x is:

$$2x + 5y = 23$$

$$2x + 15 = 23$$

$$2x = 8$$

$$x = 4$$

So the solution is $(x, y) = (4, 3)$.

Exercise 1

1 Solve the system $x + 3y = 11\ 3x + 2y = 19$ by making x the subject in the first equation, and substituting into the second equation.

2 Solve the system $4x + 3y = 16\ 3x + 5y = 1$ by making x the subject in the second equation, and substituting into the first equation.

3 Solve the system $8x - 2y = -24\ -7x + 5y = 34$ by making y the subject in the first equation, and substituting into the second equation.

2 Indices

You need to be able to use the laws of indices to simplify expressions:

$$x^a x^b = x^{a+b} \qquad \frac{x^a}{x^b} = x^{a-b} \qquad (x^a)^b = x^{ab} \qquad x^{-n} = \frac{1}{x^n}$$

Example   Simplify $\frac{(x^3x^{-1})^3x^2}{x^{12}x \div x^3}$.

$$\frac{(x^3x^{-1})^3x^2}{x^{12}x \div x^3} = \frac{(x^2)^3x^2}{x^{13} \div x^3} \quad \text{... using } x^a x^b = x^{a+b}$$

$$= \frac{x^6x^2}{x^{13} \div x^3} \quad \text{... using } (x^a)^b = x^{ab}$$

$$= \frac{x^8}{x^{13} \div x^3} \quad \text{... using } x^a x^b = x^{a+b}$$

$$= \frac{x^8}{x^{10}} \quad \text{... using } \frac{x^a}{x^b} = x^{a-b}$$

$$= x^{-2} \quad \text{... using } \frac{x^a}{x^b} = x^{a-b}$$

$$= \frac{1}{x^2} \quad \text{... using } x^{-n} = \frac{1}{x^n}$$

You need to be able to evaluate fractional and negative powers **without** using a calculator using the laws:

$$x^{\frac{1}{n}} = \sqrt[n]{x} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

Example   Evaluate $64^{-\frac{4}{3}}$.

$$64^{-\frac{4}{3}} = \frac{1}{64^{\frac{4}{3}}} \quad \text{... using } x^{-n} = \frac{1}{x^n}$$

$$= \frac{1}{\left(\sqrt[3]{64}\right)^4} \quad \text{... using } x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$$

$$= \frac{1}{4^4} \quad \text{... because } \sqrt[3]{64} = 4$$

$$= \frac{1}{256} \quad \text{... because } 4^4 = 64$$

Exercise 2

1 Simplify $\left(\frac{x^5}{x^3}\right)^7$.

2 Simplify $\left(\frac{x^9x^2}{x^5}\right)^3$.

3 Evaluate 10^{-3}.

4 Evaluate $27^{\frac{1}{3}}$.

5 Evaluate $25^{-\frac{1}{2}}$.

6 Evaluate $81^{-\frac{3}{4}}$.

3 Simplifying Surds

You need to be able to simplify surds using the rules:

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Example Write $\sqrt{50}$ in the form $k\sqrt{2}$ where k is an integer.

$\sqrt{50} = \sqrt{25 \times 2}$ ... look for two factors where one is a square number

$= \sqrt{25}\sqrt{2}$ ... using $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= 5\sqrt{2}$

Example Write $7\sqrt{20} + 2\sqrt{45}$ in the form $a\sqrt{k}$ where a and k are integers.

$7\sqrt{20} + 2\sqrt{45} = 7\sqrt{4 \times 5} + 2\sqrt{9 \times 5}$ ... look for two factors where one is a square number

$= 7\sqrt{4}\sqrt{5} + 2\sqrt{9}\sqrt{5}$ ... using $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= 7 \times 2 \times \sqrt{5} + 2 \times 3 \times \sqrt{5}$

$= 14\sqrt{5} + 6\sqrt{5}$

$= 20\sqrt{5}$

Example Evaluate $\frac{\sqrt{3}}{\sqrt{27}}$.

$\frac{\sqrt{3}}{\sqrt{27}} = \sqrt{\frac{3}{27}}$ ... using $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$= \sqrt{\frac{1}{9}}$

$= \frac{\sqrt{1}}{\sqrt{9}}$ ... using $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$= \frac{1}{3}$

Exercise 3

1 Simplify $\sqrt{72}$ fully.

2 Evaluate $\frac{\sqrt{7}}{\sqrt{28}}$.

3 Write $\sqrt{44} + \sqrt{6}\sqrt{24}$ in the form $a + b\sqrt{k}$ for integers a, b and k.

4 $4 + a\sqrt{2} = \sqrt{98} + \sqrt{m}$. What are integers a and m?

5 Simplify and evaluate $\frac{\sqrt{3}\sqrt{15}}{\sqrt{5}}$.

4 Rationalising the Denominator

You need to be able to take a fraction where the denominator is a surd (irrational), and write it as an equivalent fraction where the denominator is not a surd (rational).

<u>Example</u> Rationalise the denominator for $\frac{5}{\sqrt{7}}$.

$\frac{5}{\sqrt{7}} = \frac{5}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$ … multiply numerator and denominator by $\sqrt{7}$.

$= \frac{5\sqrt{7}}{\sqrt{7}\sqrt{7}}$

$= \frac{5\sqrt{7}}{\sqrt{49}}$ … using $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= \frac{5\sqrt{7}}{7}$

<u>Example</u> Rationalise the denominator for $\frac{3}{2\sqrt{5}}$.

$\frac{3}{2\sqrt{5}} = \frac{3}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ … multiply numerator and denominator by $\sqrt{5}$.

$= \frac{3\sqrt{5}}{2\sqrt{5}\sqrt{5}}$

$= \frac{3\sqrt{5}}{2\sqrt{25}}$ … using $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= \frac{3\sqrt{5}}{2\times5}$

$= \frac{3\sqrt{5}}{10}$

<u>Example</u> Rationalise the denominator for $\frac{4}{2+\sqrt{5}}$.

$\frac{27}{4+\sqrt{7}} = \frac{27}{4+\sqrt{7}} \times \frac{4-\sqrt{7}}{4-\sqrt{7}}$ … multiply numerator and denominator by $4-\sqrt{7}$.

$= \frac{27(4-\sqrt{7})}{(4+\sqrt{7})(4-\sqrt{7})}$

$= \frac{108-27\sqrt{7}}{16-4\sqrt{7}+4\sqrt{7}-\sqrt{7}\sqrt{7}}$ … carefully multiply out the numerator and denominator.

$= \frac{108-27\sqrt{7}}{16-2\sqrt{5}+2\sqrt{5}-\sqrt{49}}$ … using $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= \frac{108-27\sqrt{7}}{9}$ … simplify the denominator.

$= 12 - 3\sqrt{7}$ … do the division if possible

Exercise 4

1 Rationalise the denominator of $\frac{3}{2\sqrt{11}}$.

2 Rationalise the denominator of $\frac{5}{3-\sqrt{2}}$.

3 Write $\frac{12}{3-\sqrt{3}}$ in the form $a + b\sqrt{c}$ for integers a, b and c.

5 Solving Quadratic Equations

You need to be able to solve quadratic equations $ax^2 + bx + c = 0$ using three different methods: factorising, completing the square, and the quadratic formula. We will go over completing the square next year but you need to do the other two methods

Example Solve $3x^2 + 2x - 5 = 0$ using the quadratic formula.

a = +3, b = +2, c = -5 ... identify the coefficients

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ... state the quadratic formula

$= \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times -5}}{2 \times 3}$ ... substitute the coefficients

$= \frac{-2 \pm \sqrt{4 - -60}}{6}$

$= \frac{-2 \pm \sqrt{64}}{6}$

$= \frac{-2 \pm 8}{6}$

$= -\frac{10}{6}, \frac{6}{6}$ ... there are two solutions.

$= -1\frac{2}{3}, 1$ ... simplify each solution.

Example Solve $2x^2 + 7x + 3 = 0$ by factorising.

$2x^2 + 7x + 3 \quad = 0$

$(2x \ldots)(x \ldots) \quad = 0$ ... has to be a $2x$ and an x to give us $2x^2$

$(2x + 1)(x + 3) \quad = 0$ ... we need to get $+3$ at the end, check this gives us $7x$

$2x + 1 = 0$ if $x = -\frac{1}{2}$ ... use the first bracket to find the one solution

$x + 3 = 0$ if $x = -3$ ... use the second bracket to find the other solution

Exercise 5

1 Solve $x^2 + 10x + 21 = 0$ by factorising.

2 Solve $3x^2 + 17x + 10 = 0$ by factorising.

3 Solve $6x^2 - 5x - 6 = 0$ by factorising.

4 Solve $3x^2 - 3x - 2 = 0$ using the quadratic formula.

6 Rearranging formulae

Example 1

Make x the subject of $y = 2 - 5x$

Solution: Notice that in this formula the x term is negative.

	$y = 2 - 5x$	
Add $5x$ to both sides	$y + 5x = 2$	(the x term is now positive)
Subtract y from both sides	$5x = 2 - y$	
Divide both sides by 5	$x = \frac{2-y}{5}$	

Example 2

Make x the subject of $x^2 + y^2 = w^2$

Solution:	$x^2 + y^2 = w^2$	
Subtract y^2 from both sides:	$x^2 = w^2 - y^2$	(this isolates the term involving x)
Square root both sides:	$x = \pm\sqrt{w^2 - y^2}$	

Remember that you can have a positive or a negative square root. We cannot simplify the answer anv more.

Example 3 Make a the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

	$t = \frac{1}{4}\sqrt{\frac{5a}{h}}$
Multiply by 4	$4t = \sqrt{\frac{5a}{h}}$
Square both sides	$16t^2 = \frac{5a}{h}$
Multiply by h:	$16t^2 h = 5a$
Divide by 5:	$\frac{16t^2 h}{5} = a$

Example 4

Example 6: Make t the subject of the formula $a - xt = b + yt$

Solution: $a - xt = b + yt$

Start by collecting all the t terms on the right hand side:

Add xt to both sides: $a = b + yt + xt$

Now put the terms without a t on the left hand side:

Subtract b from both sides: $a - b = yt + xt$

Factorise the RHS: $a - b = t(y + x)$

Divide by $(y + x)$: $\frac{a-b}{y+x} = t$

So the required equation is $t = \frac{a-b}{y+x}$

Example 5

Make W the subject of the formula $T - W = \frac{Wa}{2b}$

his formula is complicated by the fractional term. We begin by removing the

$2b$: $2bT - 2bW = Wa$

both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by $a + 2b$: $W = \frac{2bT}{a+2b}$

Exercise 6

1. Make t the subject of the following

(a) $P = \frac{wt^2}{32r}$

(b) $V = \frac{1}{3}\pi t^2 h$

(c) $P = \sqrt{\frac{2t}{q}}$

(d) $2f^2 = \sqrt{\frac{t+2}{3}}$

2. Make the variable in brackets the subject

(a) $\frac{a(b-c)}{d} = e$ $[d]$

(b) $\frac{a(b-c)}{d} = e$ $[b]$

(c) $5m - n = 4(3m + n)$ $[m]$

(d) $y = \frac{(x+1)}{(x-1)}$ [x]

7 Solving simultaneous equations (one linear, one quadratic)

Example 1

Solve these simultaneous equations.

$$x^2 + y^2 = 5$$
$$x + y = 3$$

Call the equations (1) and (2):

$$x^2 + y^2 = 5 \quad (1)$$
$$x + y = 3 \quad (2)$$

Rearrange equation (2) to obtain:

$$x = 3 - y$$

Substitute this into equation (1), which gives:

$$(3 - y)^2 + y^2 = 5$$

Expand and rearrange into the general form of the quadratic equation:

$$9 - 6y + y^2 + y^2 = 5$$
$$2y^2 - 6y + 4 = 0$$

Cancel by 2:

$$y^2 - 3y + 2 = 0$$

Factorise:

$$(y - 1)(y - 2) = 0$$
$$\Rightarrow y = 1 \text{ or } 2$$

Substitute for y in equation (2):

When $y = 1$, $x = 2$ and when $y = 2$, $x = 1$

Note that you should always give answers as a pair of values in x and y.

Example 2

Find the solutions of the pair of simultaneous equations: $y = x^2 + x - 2$ and $y = 2x + 4$

This example is slightly different, as both equations are given in terms of y, so substituting for y gives:

$$2x + 4 = x^2 + x - 2$$

Rearranging into the general quadratic:

$$x^2 - x - 6 = 0$$

Factorising and solving gives:

$$(x + 2)(x - 3) = 0$$
$$x = -2 \text{ or } 3$$

Substituting back to find y:

When $x = -2$, $y = 0$

When $x = 3$, $y = 10$

So the solutions are $(-2, 0)$ and $(3, 10)$.

Exercise 7

a) $x^2 + y^2 = 25$
$x + y = 7$

b) $y = x^2 + 3x - 3$
$y = 2x + 1$

8 (a) Mean, median and mode from a frequency table

Find the mean of the frequency distribution below

Number	50	52	54	56	58	60	62	64	66	68	70
Frequency	2	5	4	5	15	20	15	16	10	5	3

Rewriting the frequency distribution gives

x	f	fx
50	2	2x50 = 100
52	5	5x52= 260
54	4	4x54= 216
56	5	5x56= 280
58	15	15x58=870
60	20	20x60=1200
62	15	15x62= 930
64	16	16x64=1024
66	10	10x66= 660
68	5	5x68= 340
70	3	3x70= 210

Mean = = 60.9

It can be helpful to add a cumulative frequency column to help find the median

x	f	cum freq
50	2	2
52	5	7
54	4	11
56	5	16
58	15	31
60	20	51
62	15	66
64	16	82
66	10	92
68	5	97
70	3	100

There are 100 values so the median is in between the 50^{th} and 51^{st} value.

8 (b) Calculating an estimate for the mean from a grouped frequency table

When data has been grouped it is not possible to find the actual mean of the data as we no longer have all the actual values. The mid-value of each class interval if used to represent the class as a whole and using this we can calculate an **estimate** for the mean.

Find the mean of the following frequency distribution

Class	0 - 24	25 - 49	50 - 74	75 - 99
Frequency	17	49	46	8

We need to add two new columns

Class	f	x	fx
0 – 24	17	12	17x12= 204
25 -49	49	37	49x37=1813
50 -74	46	62	46x62=2852
75 -99	8	87	8x87 =696

Mean = = 46.375

Exercise 8

1. The table below shows the number of calls per day received at a fire station over a given period.

Number of calls per day	0	1	2	3	4	5
Number of days	75	62	32	16	9	6

Calculate (a) the mean number of calls per day

(b) the median number of calls per day

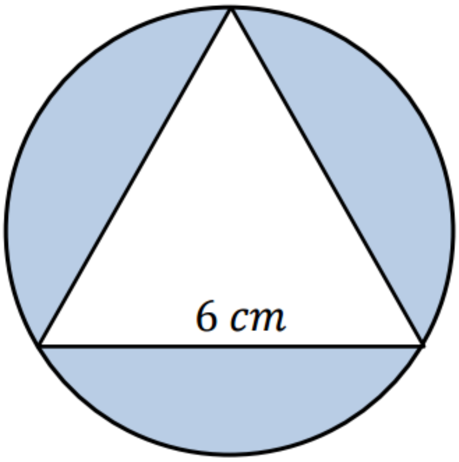
2. The marks obtained by 100 candidates were distributed as follows

Mark	0 - 19	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 99
Number of candidates	3	9	17	25	27	10	5	4

Calculate an estimate for the mean mark for these 100 candidates.

9 Problem Solving

1 This diagram shows an equilateral triangle of side length 6 cm drawn inside a circle so that each corner touches the circumference of the circle.



What area of the circle is shaded?

2

A ball is dropped and bounces up to a height that is 75% of the height from which it was dropped.

It then bounces again to a height that is 75% of the previous height and so on.

How many bounces does it make before it bounces up to less than 20% of the original height from which it was dropped?

Solutions

Just to get warmed up!

1. $3x^2 - 4x$
2. $a - 12$
3. $5pq - 9p^2$
4. $a^2 - 2b + ab$
5. $7 - x$
6. $4x - 9$
7. $-4y^2 + 24y - 10$

Exercise 1

1 $(x, y) = (5, 2)$.

2 $(x, y) = (7, -4)$.

3 $(x, y) = (-2, 4)$.

Exercise 2

1 x^{14}.

2 x^{18}.

3 $\frac{1}{1000}$.

4 3

5 $\frac{1}{5}$.

6 $\frac{1}{27}$.

Exercise 3

1 $6\sqrt{2}$

2 $\frac{1}{2}$

3 $12 + 2\sqrt{11}$

4 $a = 7\sqrt{2}, m = 16$

5 9

Exercise 4

1 $\frac{3\sqrt{11}}{22}$.

2 $\frac{15-5\sqrt{2}}{7}$.

3 $6 + 2\sqrt{3}$.

Exercise 5

1 -3, -7

2 $-\frac{2}{3}, -5$

3 $-\frac{2}{3}, \frac{3}{2}$

4 $\frac{3\pm\sqrt{33}}{6}$

Exercise 6

1. (a) $t = \sqrt{\frac{32Pr}{w}}$

   (b) $t = \sqrt{\frac{3V}{\pi h}}$

   (c) $t = \frac{p^2 q}{2}$

   (d) $t = 12f^4 - 2$

2. (a) $d = \frac{a(b-c)}{e}$

   (b) $b = \frac{de+ac}{a}$

   (c) $m = -\frac{5}{7}n$

   (d) $x = \frac{1+y}{y-1}$

Exercise 7

1. (3,4) and (4,3)
2. (2,5) and $(-2, -3)$

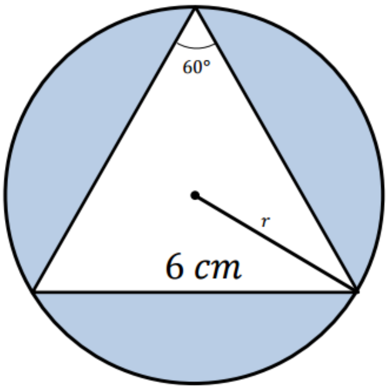
Exercise 8

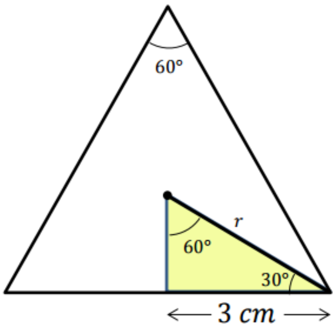
1. Mean = 1.2   Median = 1
2. Mean = 47.95

Transition Booklet Problem Solving Solutions

The triangle has internal angles of $60°$

The radius of the circle is required so marking the centre of the circle and one radius (to one of the corners of the triangle gives





There are a number of ways of proving that the angles in the marked triangle are $30°$ and 6

From this $r = \frac{3}{\cos 30}$ so $r = 2\sqrt{3}$ cm

The other side of the triangle is $\sqrt{3}$ cm.

The area of the equilateral triangle can be found by using six of the shaded right angle triar (there are plenty of other ways).

Area of equilateral triangle $= 6 \times \frac{1}{2} \times 3 \times \sqrt{3} = 9\sqrt{3}\ cm^2$

Area of circle $= \pi \times (2\sqrt{3})^2 = 12\pi\ cm^2$

So the shaded area $= 12\pi - 9\sqrt{3}\ cm^2$. This is $22.1\ cm^2$ to 1 d.p.

1. Let's call the original height the ball was dropped from h. After the first bounce we know it reached 75% of this height, so we can write this as 0.75h.

   After the next bounce it reached a height of:

   0.75 x 0.75h = 0.5625h

2. The question is asking to find the number of bounces it took for the height of the ball to be less than 20% of the original height. In other words less than 0.2h.

3. After 3 bounces, the ball reached 0.75 x 0.5625h = 0.4219h. This is only 42.19% of the original height, so it is not small enough yet.

4. After 4 bounces, the ball reached 0.75 x 0.4219h = 0.3164h. This is still not small enough.

5. After 5 bounces, the ball reached 0.75 x 0.3164h = 0.2373h. Still not small enough!

6. Finally, after 6 bounces, the ball's height was 0.75 x 0.2373h = 0.1780h, or 17.8% of its original height, which is less than 20%! So the answer is 6 bounces.